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COMPTON SCATTERING INDUCED BY RELATIVISTIC ELECTRONS

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16. Abstract

A study is made of induced Compton scattering by relativistic electrons as a process playing a role in high luminosity cosmic sources, and of the influence of such scattering on the radiation spectrum and energy losses of electrons. Consideration is confined to isotropic radiation and electron distribution and to the classic Thompson approximation.

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COMPTON SCATTERING INDUCED BY RELATIVISTIC ELECTRONS

Yu. P. Ochelkov and V. M. Charugin

Introduction

In high luminosity cosmic sources such as quasars, galactic nuclei, and pulsars, there is a high density of radiation and relativistic electrons. When these induced processes begin to play a role in this they become substantial when the brightness temperature of the radiation exceeds the kinetic temperature of the radiated electrons. The effect of induced processes in radiation in a magnetic field in the form of radiation spectra and electrons and also electron energy losses have been well studied. The generally accepted explanation of radiation spectrum bending in the low frequency area of a number of cosmic radio emission sources is that of synchrotron reabsorption of the radiation, that is, induced absorption in a magnetic field. However, where there is a high radiation density the compton scattering of photons by relativistic electrons and induced processes connected with the scattering also becomes significant. /7*

A number of papers have been written recently on induced Compton scattering. The effects of induced scattering by nonrelativistic electrons on the spectrum of the sources and electron heating have been studied in [1, 2]. It is shown that this effect may lead to a significant distortion of the first spectrum and to heating of the electrons to relativistic temperatures. The present article is devoted to a study of induced Compton scattering by relativistic electrons as one of the processes playing a role in high luminosity cosmic sources, and its influence on the radiation spectrum and energy losses of electrons.

We shall restrict our consideration to the case of isotropic distribution of radiation and electrons. We shall also confine ourselves to the classic Thompson approximation, that is, for scattering we shall consider only the Doppler shift of the photon energy, while neglecting the yield of the Doppler shift [3, 4].

*Numbers in the margins indicate pagination in the foreign text.

Scattering Probability

For induced Compton scattering it is necessary to take into account scattering of both increased energy (frequency) and of reduced photon energy. Then this scattering takes place with increased photon energy, when $\theta_2 < \theta_1$, and with reduced energy, when $\theta_2 > \theta_1$. θ_1 and θ_2 are the angles between photon impulses up to (κ) and after $(\bar{\kappa}_\rho)$ the scattering and the electron pulse $\bar{P}$. In the first case the electron loses energy and in the second it bends. In the approximation we are examining the probability of formation of a photon of energy ε_ρ for scattering by an electron of energy E of a photon with energy ε is defined by the formula

$$W(E,\varepsilon,\varepsilon_\rho) = \frac{c}{4\pi}\int (1-\beta\cos\theta)\,\sigma(\bar{\kappa},\bar{\kappa}_\rho,\bar{p})\,\delta\left(\varepsilon_\rho-\varepsilon\frac{1-\beta\cos\theta_1}{1-\beta\cos\theta_2}\right)d\Omega\, d\Omega_\rho . \qquad (1)$$

The cross-section of Compton scattering is of the form [4],

$$\sigma(\bar{\kappa},\bar{\kappa}_\rho,\bar{p}) = \frac{r_e^2}{2\gamma^2(1-\beta\cos\theta_2)^2}\left\{\left[\frac{(1-\beta\cos\theta)\,\gamma^{-2}}{(1-\beta\cos\theta_1)(1-\beta\cos\theta_2)}-1\right]^2+1\right\}.$$

where θ is the angle between pulses of the incident and scattered photons.

$$\cos\theta = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos\varphi .$$

However, it is simpler to obtain W by using the expression for this probability in the region $\varepsilon \leq \varepsilon_\rho \leq 4\gamma^2\varepsilon$ obtained in [3].

$$W(E,\varepsilon,\varepsilon_\rho) = c\,\frac{\pi r_e^2}{4}\,\frac{1}{\varepsilon\gamma^2}\left\{2\frac{\varepsilon_\rho}{\varepsilon\gamma^2}-\frac{\varepsilon_\rho^2}{\varepsilon^2\gamma^4}+4\left(\frac{\varepsilon_\rho}{\varepsilon}+1\right)\frac{1}{\gamma^2}\ln\frac{\varepsilon_\rho}{\varepsilon 4\gamma^2}+8\right\}. \qquad (2)$$

$$\gamma = \frac{E}{mc^2}$$

In fact, from the principle of partial equilibrium (the probability of the direct process is equal to the probability of the reverse process) it follows that

$$W(\bar{P},\bar{\kappa};\bar{P}_\rho,\bar{\kappa}_\rho) = W(\bar{P}_\rho,\bar{\kappa}_\rho;\bar{P},\bar{\kappa}).$$

Averaging over the angles $\Omega(\bar{p})$ and $\Omega(\bar{\kappa})$ and performing summation over all the finite states of the photon and electron with energies ε_ρ and E respectively we get

$$W(E,\varepsilon,E_\rho,\varepsilon_\rho) = \frac{1}{(4\pi)^2}\int W(\bar{P},\bar{\kappa};\bar{P}_\rho,\bar{\kappa}_\rho)\frac{dP_\rho}{dE}\,d\Omega(\bar{P})\,d\Omega(\bar{\kappa})\,d\Omega(\bar{P}_\rho)\,d\Omega(\bar{\kappa}_\rho)$$

Taking into account the law of conservation of energy for scattering and the fact that in our approximation $\varepsilon_\rho - \varepsilon \ll E$, for this probability we have /9

$$W(E,\varepsilon,\varepsilon_\rho) = W(E,\varepsilon_\rho,\varepsilon)\frac{\varepsilon_\rho^2}{\varepsilon^2}. \qquad (4)$$

The probability on the lefthand side of the equation describes scattering with decrease in the photon energy ($\varepsilon_\rho < \varepsilon$), and the righthand side incorporates expression (2) in which ε and ε_ρ change places. From (3) it follows that the total probability over the entire range of scattering frequencies is determined by the relation

$$W(E,\varepsilon,\varepsilon_\rho) = \frac{c\pi r_e^2}{4\gamma^2\varepsilon}\left[2\frac{\varepsilon_\rho}{\varepsilon\gamma^2} - \frac{\varepsilon_\rho^2}{\varepsilon^2\gamma^4} + \frac{4}{\gamma^2}\left(\frac{\varepsilon_\rho}{\varepsilon}+1\right)\ln\frac{\varepsilon_\rho}{4\varepsilon\gamma^2} + 8\right]\theta(\varepsilon_\rho-\varepsilon) +$$
$$+ c\frac{\pi r_e^2}{4\gamma^2}\frac{\varepsilon_\rho}{\varepsilon}\left[2\frac{\varepsilon}{\varepsilon_\rho\gamma^2} - \frac{\varepsilon^2}{\varepsilon_\rho^2 4\gamma^4} + \frac{4}{\gamma^2}\left(\frac{\varepsilon}{\varepsilon_\rho}+1\right)\ln\frac{\varepsilon}{4\varepsilon_\rho\gamma^2} + 8\right]\theta(\varepsilon-\varepsilon_\rho)$$
$$\frac{1}{4\gamma^2}\varepsilon \leq \varepsilon_\rho \leq 4\gamma^2\varepsilon.$$

Expression (4) satisfies the principle of partial equilibrium (3) and should be used when analyzing induced Compton processes. This expression for probability differs from that obtained in [4]. As was to be expected, the total probability equals

$$\int W(E,\varepsilon,\varepsilon_\rho)\,d\varepsilon_\rho = c\sigma_T.$$

Probability (4) vanishes for boundary values of scattered quantum energies. According to (4) the density of scattering by one radiation electron (in spontaneous scattering) is proportional to

$$\boxed{\varepsilon_\rho W(E,\varepsilon,\varepsilon_\rho) \sim \begin{cases} \varepsilon_\rho & \text{for } \varepsilon < \varepsilon_\rho \ll 4\gamma^2\varepsilon \\ \varepsilon_\rho^2 & \text{for } \frac{1}{4\gamma^2}\varepsilon \ll \varepsilon_\rho < \varepsilon \end{cases}}$$

Photon Transfer Equation

In a medium which is optically thin for Compton scattering (an area with dimensions L), $2_6 = \sigma_T NL \ll 1$, where N is the total density of the relativistic electrons, the transfer equation for photon density assumes the form [5, 6, 7, 8] /10

$$\frac{d}{dt}n(\varepsilon) = \int_0^\infty W(E,\varepsilon',\varepsilon)\,n(\varepsilon')\,N(E)\,d\varepsilon'\,dE +$$
$$+ \frac{(\hbar c)^3}{\pi\varepsilon^2}\int n(\varepsilon')\,n(\varepsilon)\,W(E,\varepsilon',\varepsilon)(\varepsilon'-\varepsilon)E^2\frac{\partial}{\partial E}\left(\frac{N(E)}{E^2}\right)d\varepsilon'\,dE. \qquad (5)$$

Here N(E) is the density of relativistic electrons per unit energy interval. $W(E,\varepsilon^1,\varepsilon)$ is determined by formula (4). The first member in (5) describes the spontaneous scattering and the second the balance of induced processes on scattering. Expression (5) may be rewritten in the form

$$\frac{d}{dt} n(\varepsilon) = j_+ - j_- + n(\varepsilon)\mu_+ + n(\varepsilon)\mu_- . \tag{6}$$

Here j_+ and j_- are coefficients of spontaneous radiation for scattering respectively for $\varepsilon > \varepsilon$ and $\varepsilon < \varepsilon^1$.[1] μ_+ and μ_- are the coefficients of oscillation and attenuation of the radiation

$$\mu_+ = \frac{(\pi \hbar c)^3}{\pi \varepsilon^2} \int_0^{\varepsilon} d\varepsilon' \int_{\frac{mc^2}{2}\sqrt{\frac{\varepsilon}{\varepsilon'}}}^{\infty} W(E,\varepsilon',\varepsilon)\, n(\varepsilon')(\varepsilon-\varepsilon') E^2 \frac{\partial}{\partial E}\left(\frac{N(E)}{E^2}\right) dE = -\frac{(\pi \hbar c)^3}{\pi \varepsilon^2} \int_0^{\varepsilon} n(\varepsilon')(\varepsilon-\varepsilon')\, d\varepsilon' \int_{\frac{mc^2}{2}\sqrt{\frac{\varepsilon}{\varepsilon'}}}^{\infty} \frac{N(E)}{E^2} \frac{\partial}{\partial E}\left[E^2 W(E,\varepsilon',\varepsilon)\right] dE . \tag{7}$$

$$\mu_- = \frac{(\pi \hbar c)^3}{\pi \varepsilon^2} \int_{\varepsilon}^{\infty} n(\varepsilon')(\varepsilon-\varepsilon')\, d\varepsilon' \int_{\frac{mc^2}{2}\sqrt{\frac{\varepsilon'}{\varepsilon}}}^{\infty} \frac{N(E)}{E^2} \frac{\partial}{\partial E}\left[E^2 W(E,\varepsilon',\varepsilon)\right] dE \tag{8}$$

$$j_+ = \int_0^{\varepsilon} n(\varepsilon')\, d\varepsilon' \int_{\frac{mc^2}{2}\sqrt{\frac{\varepsilon}{\varepsilon'}}}^{\infty} N(E)\, W(E,\varepsilon',\varepsilon)\, dE \tag{9}$$

$$j_- = \int_{\varepsilon}^{\infty} n(\varepsilon')\, d\varepsilon' \int_{\frac{mc^2}{2}\sqrt{\frac{\varepsilon'}{\varepsilon}}}^{\infty} N(E)\, W(E,\varepsilon',\varepsilon)\, dE . \tag{10}$$ /11

It is easily seen that $\frac{\partial}{\partial E}\left[E^2 W(E,\varepsilon',\varepsilon)\right] > 0$; hence μ_+ is always negative and describes radiation absorption, while μ is always positive and describes radiation amplification (see also [6]). However, it should be noted that such amplification has nothing in common with radiation oscillation (negative reabsorption), in which the electrons give up their radiation energy. In this case the electrons give up their radiation energy. In this case the electron always gains energy as a result of induced processes, as will be shown on the following page. The radiation amplification in the low energy region is explained by the fact that photons, which are bosons, have a tendency to accumulate in the states with the least energy. The coefficient $\mu = \mu_+ \pm \mu_-$ may be either positive or

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[1] The reabsorption coefficient is equal to $\frac{\mu}{c}$, and the optical thickness for for Compton reabsorption is determined by the relation $\tau = \frac{\mu}{c} L$.

negative depending upon the specific form of the distribution of the photons and electrons. Thus in [7] if we substitute $\chi = \frac{\varepsilon}{\varepsilon^1}\frac{1}{4\gamma^2}$ and in (8) $\chi = \frac{\varepsilon^1}{\varepsilon}\frac{1}{4\gamma^2}$, when we get

$$\mu = -\frac{3}{8}\sigma_T \frac{(\pi\hbar c)^3}{\pi\varepsilon}\int \frac{N(E)}{E^3}dE \int_{\frac{1}{4\gamma^2}}^{1}\Big\{\Big(1-\frac{1}{4\gamma^2 x}\Big)\Big[-3+4x+2\ln x-\frac{1}{2\gamma^2}\Big(\frac{\ln x}{x}+\frac{1}{x}\Big)\Big]\Big[n\Big(\frac{\varepsilon}{4\gamma^2 x}\Big)-n(\varepsilon 4\gamma^2 x)\Big]\Big\}dx \tag{11}$$

Since the expression in the braces is always greater than zero, then $\mu > 0$, if $n(\varepsilon)$ increases as ε increases; $\mu < 0$ if $n(\varepsilon)$ decreases as ε increases; $\mu = 0$ if $n(\varepsilon) = \mathrm{Const}$. The conclusions presented are only correct for the case in which $n(\varepsilon)$ behaves in the way indicated over the entire interval of energies $\frac{1}{4\gamma^2}\varepsilon \le \varepsilon' \le 4\gamma^2\varepsilon$ for all electron energies E.

Let there be at the initial moment monochromatic radiation with $n(\varepsilon) = n_0\delta(\varepsilon-\varepsilon_0)$; then the solution to equation (6) may be written in the form

$$\begin{aligned} n(\varepsilon) &= \frac{J_0}{|\mu_+|}\left(1-e^{-|\mu_+|t}\right) && \text{for } \varepsilon > \varepsilon_0; \\ n(\varepsilon) &= \frac{J_0}{|\mu_-|}\left(e^{|\mu_-|t}-1\right) && \text{for } \varepsilon < \varepsilon_0. \end{aligned} \tag{12}$$

In the optically thin case $/\mu/\tau \ll 1$ /12

$$\begin{aligned} n(\varepsilon) &= J_0 t \sim \mathrm{Const} && \text{for } \varepsilon_0 < \varepsilon \ll 4\gamma^2\varepsilon_0, \\ n(\varepsilon) &= J_0 t \sim \varepsilon && \text{for } \frac{1}{4\gamma^2}\varepsilon_0 \ll \varepsilon < \varepsilon_0. \end{aligned}$$

In the optically dense case $/\mu/\tau \gg 1$ the solution has the form

$$\begin{aligned} n(\varepsilon) &= \frac{J_0}{|\mu_+|} && \text{for } \varepsilon > \varepsilon_0, \\ n(\varepsilon) &= \frac{J_0}{|\mu_-|}e^{|\mu_-|t} && \text{for } \varepsilon < \varepsilon_0. \end{aligned}$$

In order to represent the solution of (12) clearly, let us consider the case of the thermal spectrum of electrons of temperature τ_e

$$N(E) = AE^2 e^{-\frac{E}{\kappa T_e}}$$

By using (6), (7), (8), (9), (10) it is easy to show that

$$\begin{aligned} \mu_+ &= -\frac{(\pi\hbar c)^3}{\pi\varepsilon^2}(\varepsilon-\varepsilon_0)\frac{J_0}{\kappa T_e}, \\ \mu_- &= \frac{(\pi\hbar c)^3}{\pi\varepsilon}(\varepsilon_0-\varepsilon)\frac{J_0}{\kappa T_e}. \end{aligned} \tag{13}$$

Consequently, in an optically dense area we have

$$n(\varepsilon) = \frac{J_0}{|\mu_+|} = \frac{\pi\varepsilon}{(\pi\hbar c)^3}\kappa T_e \quad \text{for } \varepsilon > \varepsilon_0.$$

This coincides with the Rayleigh-Jeans formula for thermal radiation with a temperature equal to the temperature of the electrons. Thus induced scattering leads the system of radiation and relativistic electrons toward thermal equilibrium [2, 4, 5, 9, 10].

For monoenergetic distribution of electrons $N(E) = N\delta(F - E_0)$ the Compton scattering in the optically dense case and for $\varepsilon \gg \varepsilon_0$ yields the following radiation picture (the subscript zero for electrons is omitted below):

$$J_+ = \frac{3}{4} c\sigma_T N n \frac{1}{\varepsilon_0 \gamma^2}\left\{x - 2x^2 + \frac{1}{2\gamma^2}\ln x + 2x\ln x + 1\right\}\Theta\left(x - \frac{1}{4\gamma^2}\right) = \frac{3}{4} c\sigma_T N n \frac{1}{\gamma^2\varepsilon_0} = \frac{3}{4}\frac{c}{L}\tau n \frac{1}{\gamma^2 \varepsilon_0}; \tag{14}$$

$$N_+ = \frac{3}{2} c\sigma_T N n \frac{1}{\gamma^2\varepsilon_0}\frac{(\pi\hbar c)^3}{\varepsilon\varepsilon E}\left\{-3x+4x^2-2x\ln x - \frac{1}{2\gamma^2}(\ln x+1)\right\}\left(1-\frac{1}{4\gamma^2 x}\right) = \frac{3}{8}\frac{c}{L}\tau_c \frac{\kappa T_b}{E}\left(\frac{mc^2}{\varepsilon}\right)^4\left(2\ln\frac{\varepsilon_0 4\gamma^2}{\varepsilon}-3\right)$$

/13

τ_b is the brightness temperature of radiation with energy ε_0. In the latter parts of equation (14) the exact expression for $\varepsilon_0 \lesssim \varepsilon$ is substituted for the asymptotic one when

$$n(\varepsilon) = \frac{J_+}{N_+} = \frac{1}{2}\frac{\varepsilon\varepsilon E}{(\pi\hbar c)^3}\frac{\left(x-2x^2+\frac{1}{2\gamma^2}\ln x+2x\ln x+1\right)}{\left(-3x+4x^2-2x\ln x-\frac{1}{2\gamma^2}(\ln x+1)\right)\left(1-\frac{1}{4\gamma^2 x}\right)}$$

The brightness temperature of the radiation in the region $\varepsilon > \varepsilon_0$ is equal to

$$\kappa T_b = \frac{E}{2}\frac{\left(x-2x^2+\frac{1}{2\gamma^2}\ln x+2x\ln x+1\right)}{\left(-3x+4x^2-2x\ln x-\frac{1}{2\gamma^2}(\ln x+1)\right)\left(1-\frac{1}{4\gamma^2 x}\right)}$$

In the two limiting cases we have

$$\kappa T_b \approx \begin{cases} \frac{E}{2}. & \text{for } \varepsilon \simeq 4\gamma^2\varepsilon_0, \\ \frac{E}{2}\frac{\varepsilon_0}{\varepsilon}\frac{4\gamma^2}{\left(2\ln\frac{\varepsilon_0 4\gamma^2}{\varepsilon}-3\right)} \gg \frac{E}{2} & \text{for } \varepsilon_0 \ll \varepsilon \ll 4\gamma^2\varepsilon_0. \end{cases}$$

In the latter case the radiation brightness temperature is inversely proportional to the photon energy, that is, as in the optically thin case the radiation intensity in the range $\varepsilon > \varepsilon_0$ is proportional to the radiation frequency. This result differs radically from the results for sources with synchroton reabsorption, in which for monoenergetic distribution of electrons, as for Maxwellian distribution, the intensity of radiation in optically dense areas has a Rayleigh-Jeans spectrum with a temperature $T_б \simeq E$. Analogous intensity calculations in energy region $\varepsilon < \varepsilon_0$ yield

$$n(\varepsilon) = \frac{1}{|\mu_-|}\left(e^{|\mu_-|}-1\right) \quad \text{for } \varepsilon \gg \frac{\varepsilon_0}{4\gamma^2}$$

$$J_- = \frac{3}{4}\frac{c}{L}\tau_e\, n\, \frac{\varepsilon}{\varepsilon_0\gamma^2}$$

$$\mu_- = \begin{cases} \frac{3}{8}\frac{c}{L}\tau_e\left(\frac{mc^2}{\varepsilon}\right)^5\frac{\kappa T_e}{mc^2}\left\{2\ln\frac{\varepsilon}{\varepsilon_0}4\gamma^2-3\right\}\left(\frac{\varepsilon_0}{\varepsilon}\right)^2 & \text{for } \varepsilon \gg \frac{\varepsilon_0}{4\gamma^2} \\ \frac{3}{8}\frac{c}{L}\tau_e\left(\frac{mc^2}{\varepsilon}\right)^5\frac{\kappa T_e}{mc^2}\left\{2\ln\frac{\varepsilon}{\varepsilon_0}4\gamma^2-3\right\}\frac{\varepsilon_0-\varepsilon}{\varepsilon_0} & \text{for } \varepsilon_0-\varepsilon \ll \varepsilon \\ 6\frac{c}{L}\tau_e\frac{\kappa T_e}{\varepsilon} & \text{for } \varepsilon \simeq \frac{\varepsilon_0}{4\gamma^2} \end{cases}$$

The maximum value of μ_- is reached when $\varepsilon = \frac{\varepsilon_0}{4\gamma^2}$, as ε and $\mu_- \sim \varepsilon^{-2}$. /14

Under real astrophysical conditions the distribution of electrons may be approximated by the following expression [6]:

$$N(E) = \begin{cases} KE^2 & \text{for } E < E_0 \\ KE_0^{\gamma+2}E^{-\gamma} & \text{for } E > E_0 \end{cases}$$

For such a distribution we have

$$\mu = \begin{cases} -9F(\gamma)\frac{(\gamma-1)}{(\gamma+2)}\frac{c}{L}\tau_e\left(\frac{mc^2}{E_0}\right)^{-\gamma}\left(\frac{\kappa T_e}{E_0}\right)2^{\gamma}\left(\frac{\varepsilon}{\varepsilon_0}\right)^{-\frac{\gamma+4}{2}} & \text{for } \varepsilon \gg 4\varepsilon_0\left(\frac{E_0}{mc^2}\right)^2 \\ -\frac{9}{4}\frac{(\gamma-1)}{(\gamma+2)}\frac{c}{L}\tau_e\left(\frac{mc^2}{E_0}\right)^2\frac{\varepsilon_0(\varepsilon-\varepsilon_0)}{\varepsilon^2}\left(\frac{\kappa T_e}{E_0}\right) & \text{for } \varepsilon_0 < \varepsilon \ll 4\varepsilon_0\left(\frac{E_0}{mc^2}\right)^2 \\ \frac{9}{4}\frac{(\gamma-1)}{(\gamma+2)}\frac{c}{L}\tau_e\left(\frac{mc^2}{E_0}\right)^2\frac{(\varepsilon_0-\varepsilon)}{\varepsilon}\left(\frac{\kappa T_e}{E_0}\right)\ln\frac{1}{\varepsilon} & \text{for } \frac{\varepsilon_0}{4}\left(\frac{mc^2}{E_0}\right)^2 \ll \varepsilon < \varepsilon_0 \\ 9\frac{(\gamma-1)}{(\gamma+2)}F(\gamma)\frac{c}{L}\tau_e\left(\frac{mc^2}{E_0}\right)^{-\gamma}\frac{(\varepsilon_0-\varepsilon)}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon_0}\right)^{\frac{\gamma+2}{2}}\left(\frac{\kappa T_e}{E_0}\right)2^{\gamma} & \text{for } \varepsilon \ll \frac{\varepsilon_0}{4}\left(\frac{mc^2}{E_0}\right)^2 \end{cases}$$

$F(\gamma) \sim 1$ is the spectral index function. From the formulas obtained it is evident that for photons of energies $\varepsilon_0 \lesssim \varepsilon \ll 4\varepsilon_0\left(\frac{E_0}{mc^2}\right)^2$ and $\frac{\varepsilon_0}{4}\left(\frac{mc^2}{E_0}\right)^2 \ll \varepsilon < \varepsilon_0$ μ_+ and μ_- differ from the expressions for the thermal spectrum (13) only in the numerical factor. $|\mu_+|$ has the maximum value for $\varepsilon \sim \varepsilon_0$, which equals

$$|\mu_+|_{max} \simeq \frac{9}{4}\frac{(\gamma-1)}{(\gamma+2)}\frac{c}{L}\tau_e\left(\frac{mc^2}{E_0}\right)^2\frac{\kappa T_e}{E_0}, \quad (15)$$

μ_- coincides in absolute value with μ_+ in the region $\varepsilon \sim \varepsilon_0$ and has the maximum value when

$$(\mu_-)_{max} \simeq 9\frac{(\gamma-1)}{(\gamma+2)}\frac{c}{L}\tau_e\frac{\kappa T_e}{E_0}. \quad (16)$$

μ_- rapidly decreases at lower photon energies. /15

The radiation intensity in the optically dense case for $\varepsilon_0 \lesssim \varepsilon \lesssim 4\varepsilon_0\left(\frac{E_0}{mc^2}\right)^2$ coincides within the accuracy of the numerical factor with the thermal radiation intensity.

As was to be expected, in energy region $\varepsilon \gg 4\varepsilon_0 (E/mc^2)^2$ the spectral density of photons $n(\varepsilon) \sim \varepsilon^{-3/2}$ (intensity $\sim \varepsilon n(\varepsilon) \sim \varepsilon^{-1/2}$), as for synchroton radiation. If the electron spectrum has a break for energies $E = E_*$, that is, $N(E) = C$ when $E < E_*$, as is very often assumed in analyzing radiation mechanisms, then the radiation intensity in the low energy region $\varepsilon < \varepsilon_0 4 (E_*/mc^2)^2$ would be proportional to ε, and not to ε^2.

The energy loss of electrons for induced Compton scattering may be obtained if expression (5) is multiplied and integrated with respect to ε over the entire admissible interval of photon energy. In this case we must set $N(E) = \delta(E - E_0)$.

$$\left(\frac{dE}{dt}\right)_c = P_c = -c\sigma_T \rho \frac{4}{3}\left(\frac{E}{mc^2}\right)^2 + \frac{(\pi\hbar c)^3}{\delta E^2}\int \frac{n(\varepsilon)\, n(\varepsilon')}{\varepsilon^2}(\varepsilon' - \varepsilon)^2 \frac{\partial}{\partial E}\left[E^2 W(E, \varepsilon', \varepsilon)\right] \delta(\varepsilon' - \varepsilon)\, d\varepsilon\, d\varepsilon'$$

The first term on the right describes the spontaneous Compton losses; quanta of energy

$$\varepsilon_p \simeq \varepsilon \left(\frac{E}{mc^2}\right)^2$$

are formed mainly in this process. The second term describes losses for induced scattering. The characteristic energies of scattered quanta are determined by the interval of photon energies, in which it is necessary to take induced scattering into account. In deriving (17) we took into account the properties of the cross-section (3) and final expression (17) contains probability (2) describing scattering with increase in the energy of the scattered photons. As is to be seen from (17) in the isotropic case $P_c^{ind} > 0$ for any forms of photon /16
distribution. In fact since $\frac{1}{4\gamma^2} \leq x \leq 1$, then

$$\frac{\partial}{\partial E}\left[E^2 W(E, \varepsilon, \varepsilon')\right] = 2\frac{3}{2}\sigma_T\left(\frac{mc^2}{E}\right)\frac{mc^2}{\varepsilon'}\left\{-3x + 4x^2 - 2 + \ln x - \frac{1}{2\gamma^2}(\ln x + 1)\right\} > 0 \qquad (18)$$

Because of this in the isotropic case negative reabsorption, which is characterized by transfer of energy from radiation electrons, is impossible. Therefore the statement that such reabsorption is possible for N(F) increasing faster than E^2 [5] is untrue. The situation described above of Compton reabsorption is completely analogous to that which was discussed at one time in analysis of negative reabsorption in a vacuum for synchrotron radiation [12].

In astrophysical applications the most interesting case is that of two types of photon distribution: 1) $\rho(\varepsilon) \sim \varepsilon^{-\alpha}$ for $\varepsilon_1 \leq \varepsilon \leq \varepsilon_2$, the high brightness temperatures (higher kinetic electron temperatures) being observed in a comparatively narrow energy interval, at least $\varepsilon_2 - \varepsilon_1 << \gamma^2 \varepsilon_2$; 2) the distribution of photons in the form of two lines $\rho(\varepsilon) = \beta_1 \delta(\varepsilon - \varepsilon_1) + \beta_2 \delta(\varepsilon - \varepsilon_2)$, where the lines may be considered to be two narrow spectral areas the distance between which is much greater than the width of these areas.

1) In the first case there is basically scattering on change in photon energy $\Delta\varepsilon \sim \varepsilon$. The expression for induced losses (induced heating) may be obtained directly by substituting (2) in expression (17). But in the present approximation probability (2) assumes the form [13]

$$W(E, \varepsilon, \varepsilon_p) = \frac{3}{4} c \sigma_T \frac{1}{\gamma^2 \varepsilon} \left[1 - \frac{1}{\beta^2} \ln \frac{4\varepsilon \gamma^2}{\varepsilon_p}\right].$$

If the energy interval is comparatively wide $\Delta = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1} \gtrsim 1$ the heating is determined by the formula

$$\boxed{P_c^{ind} \simeq f(\alpha) \frac{\kappa T_e}{mc^2} c \sigma_T \rho \left(\frac{mc^2}{E}\right)^5 \ln 2\left(\frac{E}{mc^2}\right).} \tag{19}$$

$f(\alpha) \sim \alpha - 4$ is the spectral index function. If the energy interval is narrow, this being characteristic of lines $\Delta << 1$, then expression (19) assumes the form /17

$$P_c^{ind} = f(\alpha) \Delta^2 \frac{\kappa T_e}{mc^2} c \sigma_T \rho \gamma^{-5} \ln 2\gamma.$$

Such a function of Δ is understandable, since in the ideal case induced processes play no role in monochromatic waves [1]. The typical quantity determining the efficiency of induced Compton electron heating is $\varkappa$ -- the ratio of the induced heating rate to the spontaneous Compton losses:

$$\varkappa = \left|\frac{P_c^{ind}}{P_c^{sp}}\right| = f(\alpha) \frac{\kappa T_e}{mc^2} \gamma^{-7} \ln 2\gamma. \tag{20}$$

In this formula it is evident that the efficiency of induced Compton heating decreases rapidly with increase in the electron energy. Even in the most intensive sources, in which the value of the radiation brightness temperature T_e is great, the maximum value of the energy to which electrons may be heated is determined by the value of ($\varkappa \leq 1$):

$$E \lesssim mc^2 \left(\frac{\kappa T_e}{mc^2}\right)^{1/7}. \tag{21}$$

For higher electron energies the spontaneous Compton losses rapidly reduce the energy of the electrons to this value. For narrow spectral lines formula (20) must be multiplied by Δ^2, and in (21) the factor Δ^2 should be introduced at the parenthesis.

2) In the second case expression (17) is easily integrated

$$P_c^{ind} = \frac{3}{2} c \rho_1 \rho_2 \frac{(\pi \hbar c)^3}{\pi \varepsilon_1^2} \sigma_T \frac{(\varepsilon_2 - \varepsilon_1)^2}{\varepsilon_2^3} \left(\frac{mc^2}{E}\right)^2 \frac{1}{E} \left\{ -3x + 4x^2 - 2x \ln x - \frac{1}{2\gamma^2} (\ln x + 1) \right\}$$

$$x = \frac{\varepsilon_2}{\varepsilon_1} \frac{1}{4\gamma^2};$$

When $\chi \ll 1$ we get

$$P_*^{ind} \sim \rho_1 \rho_2 \frac{(\varepsilon_2 - \varepsilon_1)^2}{\varepsilon_1^3 \varepsilon_2^2} \frac{(\pi \hbar c)^3}{mc^2} c \sigma_T \left(\frac{mc^2}{E}\right)^5. \qquad (22)$$

The clearest and most interesting case is that in which $\rho_1 > \rho_2$ and $\chi \gtrsim 1$ ($\varepsilon_2 \gtrsim 4\gamma^2 \varepsilon_1$). In this case the spontaneous losses are determined by the losses arising in the scattering of photons from energy region ε_1. The total energy losses, both transitions being taken into account, are determined by the relation /18

$$P_c = -\frac{4}{3} c \sigma_T (\rho_1 + \rho_2) \left(\frac{E}{mc^2}\right)^2 + 24 c \sigma_T \rho_1 \left(\frac{E}{mc^2}\right)^2 \frac{\kappa T_2}{E} \simeq -\frac{4}{3} c \sigma_T \rho_1 \left(\frac{E}{mc^2}\right)^2 \left(1 - 18 \frac{\kappa T_2}{E}\right)$$

$$P_c^{ind} = 24 c \sigma_T \rho_1 \left(\frac{E}{mc^2}\right)^2 \frac{\kappa T_2}{E}. \qquad (23)$$

τ_2 is the radiation brightness temperature in the photon energy region. As is evident from this expression, the efficiency of induced heating is more significant than in case 1.

$$\varkappa = 18 \frac{\kappa T_2}{E} \frac{\rho_1}{\rho_1 + \rho_2} \simeq \begin{cases} \frac{\kappa T_2}{E} 18 & \text{for } \rho_1 > \rho_2 \\ \frac{\kappa T_2}{E} \frac{\rho_1}{\rho_2} 18 & \text{for } \rho_2 > \rho_1 \end{cases} \qquad (24)$$

It may be said that in this case the process of establishment of equilibrium is most effective when there is great variation in the photon energy. At equilibrium the electrons neither lose nor gain energy, the electron energy being of the order $E \simeq \kappa\tau_2$. After rapid establishment of equilibrium between the electrons and the radiation in the second line (in the second spectral interval) a slow process of establishing equilibrium between electrons and radiation with small photon energy variation takes place inside the first spectral interval. (This is of course a simplified qualitative picture of the phenomenon.)

From the astrophysical point of view it is interesting to evaluate the role of induced Compton scattering by relativistic electrons in certain cosmic sources. The characteristic quantities in this sense are the values of the optical thickness for Compton reabsorption as determined by formulas (15) and (16), and also quantity $\varkappa$ determined from (20) or (24). The optical thickness value determines the distortion of the source spectra; if $\tau > 1$, then the /19 distortions of the spectra due to induced Compton scattering may be neglected. For the distortion to be significant it is necessary that

$$N \gg \frac{E}{\kappa T_e} \frac{1}{\sigma_T L} \qquad (25)$$

The spectral distortions may obviously be significant even at small values of $\varkappa$ in sufficiently dense sources.

In quasars and quasar-like phenomena the most intensive region of infrared and submillimeter radiation is characterized by brightness temperatures $\kappa T_e \simeq (10 \div 10^3)mc^2$ (for sources of dimensions determined from observed variations of optical and infrared radiation, $L \leq 10^{16}$ cm). In spite of such high brightness temperatures, $\varkappa \ll 1$, and the influence of induced heating of electrons on the scattering of electrons will be disregarded. The question of whether induced Compton scattering by relativistic electrons produces any distortions of the source spectra depends, of course, on the density of the relativistic electrons. Using (15) and (16) we have for $\gamma = 2 \div 5$, $E_* \simeq 10^2 mc^2$:

$$\tau_+ = \frac{|N_1| L}{c} \simeq \tau_c \left(\frac{mc^2}{E_*}\right)^2 \frac{\kappa T_e}{E_*} \simeq 10^{-3} \tau_c \ll 1,$$
$$\tau_- = \tau_c \left(\frac{\kappa T_e}{E_*}\right) \simeq 10 \tau_c.$$

If we assume that $0.1 < \tau_c < 1$, then it is entirely reasonable to assume that $\tau_- > 1$. We have seen that this maximum value of optical thickness is reach-d at photon energies $\varepsilon_\rho \sim \varepsilon \frac{1}{4}{}_*{}^2$, corresponding in our case to a frequency of approximately $\sim 10^9$ Hz. Obviously these estimates will vary substantially in favor of the presence of distortions due to induce Compton scattering, if it is assumed that $E_* = 10\ mc^2$, $\tau_+ \sim \tau_c \lesssim 1$, $\tau_- \simeq (10^3 \div 10^4)\tau_c$.

Consequently, if in quasars the density of relativistic electrons with energies /20 $\lesssim 10\ mc^2$ exceeds 10^7 to 10^8 cm^3, then we should observe significant distortion of the spectra due to induced Compton scattering. The value and range of the frequencies in which these distortions should be observed depend on the specific quasar model.

In OH and H_2O maser radiation sources the brightness temperature of the radiation reaches values of $T_e \simeq 10^{14} - 10^{16}$. In spite of this the efficiency of Compton heating $\varkappa << 1$, since the value of Δ is small (because of the narrowness of the maser radiation lines).

In pulsars, in which the radiation brightness temperature exceeds the temperature of maser sources by ten orders of magnitude, > 1, up to electron energies $\mathcal{E} \lesssim (10^2 - 10^3)mc^2$. In this instance (for example, the pulsar in Crab Nebula PO 532), we have a case in which radiation is present with high brightness temperatures at the radio and optical frequencies $(\kappa T_{br} \simeq (10^2 - 10^3) mc^2)$; this apparently may assure rapid heating of the electrons to these temperatures. Moreover, spectral distortions should be observed for pulsars in a number of models [14] $\tau_- > 1$. It is true that the treatment presented for pulsars is purely qualitative in nature since in pulsars it is necessary to take into account the sharp anisotropy of radiation.

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